**MATH 32B**
**FINAL EXAMINATION**

**Please note:** Show your work. Except on true/false problems, correct answers not accompanied by sufficent explanations will receive little or no credit. Please call one of the proctors if you have any questions about a problem. No calculators, computers, PDAs, cell phones, or other devices will be permitted.

| #1 | #2 | #3 | #4 | #5 | #6 | |
|---|---|---|---|---|---|---|
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| #7 | #8 | #9 | | | | Total |
| | | | | | | |

Your section meets (circle): Tuesday Thursday

Student ID:_____

Name:_____

Signature:_____

*By signing above I certify that I am the person whose name and student ID appears on this page.*

**Problem 1.** True or false? (*Correct answer is worth 2 points; no answer is 1 point; wrong answer is 0 points).*

(1) $\mathbf{F} \cdot \text{curl}\mathbf{F} = 0$ for any $\mathbf{F}$, where $\mathbf{F}$ is a vector field, and $\cdot$ is the dot product.
(2) If $\text{div}(\mathbf{F})$ is not identically 0, then the vector field can not be conservative.
(3) If a point $P$ is the center of mass of a body $S$, then $P \in S$.
(4) If $\mathbf{F} = \frac{\mathbf{r}}{|\mathbf{r}|^3}$, then both $\text{div}(\mathbf{F}) = 0$ and $\text{curl}(\mathbf{F})=0$.
(5) $\text{div}(\text{grad}(f)) = 0$ for any differentiable function $f$.

**Problem 2.** Sketch the region of the plane whose area is given by the integral $\int_0^a \int_x^{\sqrt{2a^2-x^2}} dydx$. Change the order of integration and compute the area.

**Problem 3.** Find the area bounded by the curves $xy = a^2/2$; $xy = 2a^2$; $y = x/2$ and $y = 2x$. (*Hint:* use coordinates $(u, v)$ so that $xy = u$ and $y = vx$).

**Problem 4.** Find the volume of the solid bounded by $z = x^2 + y^2$, $x + y = 4$, $x = 0$, $y = 0$,$z = 0$.

**Problem 5.** The force field on the plane is given by $\mathbf{F} = (x+y)\mathbf{i} + 2x\mathbf{j}$. Find the total work done by this force in moving a unit mass around the unit circle with center at the origin.

**Problem 6.** Use Green's formula to compute the integral $\int_C \frac{dx}{y} - \frac{dy}{x}$, where $C$ is the triangle with vertices $A = (1, 1)$, $B = (2, 1)$ and $C = (2, 2)$, oriented from $A$ to $B$ and to $C$.

**Problem 7.** Use Stokes' theorem to compute the integral $\int_C((z-y)dx+(x-z)dy+(y-x)dz$, where the contour is the triangle with vertices $(1,0,0)$, $(0,1,0)$, $(0,0,1)$ (and orientation of the contour corresponds to the given order of points). (*Note:* Please, do NOT compute the line integral directly. There will be no points given of solutions of this type).

**Problem 8.** Find the area of the part of the surface $2z = x^2$ bounded by the planes $y = x/2$. $y = 2x$,$x = 2\sqrt{2}$.

**Problem 9.** Let $\mathbf{F} = \varepsilon Q \cdot \frac{\mathbf{r}}{|\mathbf{r}|^3}$ be the field of the electric charge $Q$ positioned at the origin of the coordinate system.

(1) Without finding the potential, show that this electric field is conservative.

(2) Show that the function $f(\mathbf{r}) = \varepsilon Q \cdot \frac{1}{|\mathbf{r}|}$ is a potential for this field.

(3) Compute the intergral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $C$ is the spiral $\mathbf{r}(t) = \cos(t)\ \mathbf{i} + \sin(t)\ \mathbf{j} + t\ \mathbf{k}$, $0 \leq t \leq 2\pi$.