5. For $1 < p < \infty$, let $\{f_n\}$ be a sequence in $L^p(\mathbb{R}^n, dx)$. Suppose that $\|f_n\|_p \leq 1$ for all n and that there exists a continuous function $h(x)$ on $\mathbb{R}^n$ such that $|f_n(x)| \leq h(x)$ for all n and almost every $x \in \mathbb{R}^n$. Show that if $f_n(x) \to 0$ for almost every $x \in \mathbb{R}^n$, then $f_n \to 0$ weakly in L^p.

6. (a) Determine all $T \in \mathcal{D}'(\mathbb{R})$ (the space of distributions on $\mathbb{R}$) satisfying the equation $xT = 0$.

(b) Show that if $S \in \mathcal{D}'(\mathbb{R})$ there is $T \in \mathcal{D}'(\mathbb{R})$ satisfying $xT = S$.

7. Let $E \subset \mathbb{R}$ be Lebesgue measurable with $0 < m(E) < \infty$. Show that for any α, $0 < \alpha < 1$, there is an open interval I such that $m(E \cap I) > \alpha m(I)$.

8. Let $L^1_{\mathbb{R}}$ be the real Banach subspace of $L^1(\mathbb{R}, dx)$ consisting of those functions which are real-valued (with its usual norm).
(a) Show that the mapping $\phi : L^1_{\mathbb{R}} \to \mathbb{R}$ given by

$$\phi(f) = \int_{\mathbb{R}} \sin\big(f(x)\big)\,dx$$

is well defined and uniformly continuous.
(b) For a fixed $f \in L^1_{\mathbb{R}}$ show that the function $\mathbb{R} \ni t \mapsto \phi(tf) \in \mathbb{R}$ is continuously differentiable.