3. (30 pts.) Let μ be counting measure on $\mathbb{N}$, the set of natural numbers, and $\{f_n\}$ a sequence of real-valued functions on $\mathbb{N}$. Show that $f_n \to f$ in measure if and only if $f_n \to f$ uniformly.

4. (30 pts.)
   (a) Show that any compact metric space has a countable dense subset.
   (b) Give an example of a metric space with no countable dense subset.

5. (30 pts.) If X is a normed linear space over $\mathbb{R}$ which has a countably infinite algebraic basis $\{x_1, x_2, \dots\}$ (and hence is of countably infinite dimension) then X is not complete. (You may assume without proof that any finite dimensional subspace of a normed linear space is closed.)

Hint: Consider the subsets $E_k = \{\sum_{i=1}^{k} a_i x_i : a_i \in \mathbb{R}\}$.