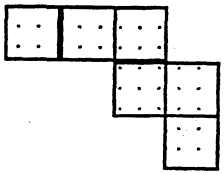
PART II

SYMMETRIC FUNCTION METHODS

S.1 Let $p_1 = x_1 + x_2 + x_3$, $p_2 = x_1^2 + x_2^2 + x_3^2$, $p_3 = x_1^3 + x_2^3 + x_3^3$ and $p_5 = x_1^5 + x_2^5 + x_3^5$. Construct the polynomial $Q(y_1, y_2, y_3)$ which expresses p_5 in terms of p_1, p_2, p_3.

S.2 Let D denote the following diagram



Give the expansion of S_D in terms of

(a) The Schur basis (using the Littlewood-Richardson rule)

(b) The elementary basis

(c) The monomial basis

S.3

(a) Compute the symmetric function

$$P(x) = \frac{1}{6!} \sum_{\sigma \in S_6} \chi^{222}(\sigma) \chi^{222}(\sigma)\, p_{\lambda(\sigma)}(x)$$

where $p_{\lambda(\sigma)}(x)$ denotes the power basis element indexed by the partition giving the cycle structure of σ.

(b) Give the representation theoretical interpretation of $P(x)$..

S.4 Let B be the representation obtained by inducing from $S_{\{1,2,3\}} \times S_{\{4,5,6,7\}}$ to S_7 the outer tensor product of the irreducible representations $A^{2,1}$ and $A^{2,2}$.

(a) Give the Frobenius image of the character of B.

(b) Determine the multiplicities of the irreps of S_7 in B.

S.5 Use the slinky rule to express the bialternant

$$\frac{\Delta_{216832}(x_1, x_2, x_3, x_4, x_5, x_6)}{\Delta_{543210}(x_1, x_2, x_3, x_4, x_5, x_6)}$$

as a signed Schur function, where $\Delta_p(x_1, x_2, \ldots, x_n) = det \| x_i^{p_j + n - j} \|_{i,j=1}^n$.