3) (30 pts.) A^λ denote the

(a) Use the Murnaghan-Nakayama rule to compute the value of the character $\chi^{(2,3)}$ at the conjugacy classes of S_5.

(b) Find the character table for $S_2 \times S_2 \times S_1$ where $S_2 \times S_2 \times S_1$ is the Young subgroup of S_5 consisting of all permuations $\sigma \in S_5$ such that

$$\sigma(1), \sigma(2) \in \{1,2\}, \sigma(3), \sigma(4) \in \{3,4\}, \sigma(5) = 5.$$

(c) Decompose $A^{(2,3)} \downarrow^{S_5}_{S_2 \times S_2 \times S_1}$ as an sum of irrecible representations of $S_2 \times S_2 \times S_1$.

4) (40 pts.)

(a) Let $A^{(1,3)} \times A^{(2,2)}$ denote the representation of $S_4 \times S_4$ such that for all $(\sigma, \tau) \in S_4 \times S_4$

$$A^{(1,3)} \times A^{(2,2)}(\sigma, \tau) = A^{(1,3)}(\sigma) \otimes A^{(2,2)}(\tau)$$

where for any matrices A and B, $A \otimes B$ denotes the tensor product of A and B. Decompose $A^{(1,3)} \times A^{(2,2)} \uparrow^{S_8}_{S_4 \times S_4}$ as a sum of irreducible representations of S_7.

(b) Show that $\{A^\lambda \times A^\mu : \lambda \vdash 4 \text{ and } \mu \vdash 4\}$ is a complete set of representatives of the irreducible representations of $S_4 \times S_4$ where $A^\lambda \times A^\mu(\sigma, \tau) = A^\lambda(\sigma) \otimes A^\mu(\tau)$.

Note: For parts (a) and (b) above, regard $S_4 \times S_4$ as a subgroup of S_8 by letting

$$S_4 \times S_4 = \{\sigma \in S_8 : \sigma(1), \sigma(2), \sigma(3), \sigma(4) \in \{1,2,3,4\}, \sigma(5), \sigma(6), \sigma(7), \sigma(8) \in \{5,6,7,8\}\}.$$

(c) Let *Alt* denote the alternating representation. Decompose $Alt \uparrow^{S_7}_{S_1 \times S_3 \times S_3}$ as a sum of irreducible representations of S_7 where $S_1 \times S_3 \times S_3$ is the Young subgroup of S_7 consisting of all permuations $\sigma \in S_7$ such that

$$\sigma(1) = 1, \sigma(2), \sigma(3), \sigma(4) \in \{2,3,4\}, \sigma(5), \sigma(6), \sigma(7) \in \{5,6,7\}.$$

(d) Decompose $A^{(2,4)} \otimes A^{(3,3)}$ as a sum of irreducible representations of S_6 where $\otimes$ represents the Kronecker product of the representations.

5) Let S_4 denote the symmetric group on 4 elements and A_4 denote the alternating group, i.e. $A_4 = \{\sigma \in S_4 : sign(\sigma) = 1\}$.

(a) Find the conjugacy classes of A_4.

(b) Let $D = \{\epsilon, (1,2)(3,4), (1,3)(2,4), (1,4)(2,3)\}$. Show that D is a normal subgroup of A_4 and that A_4/D is isomorphic to Z_3.

(c) Give the character table for Z_3.

(d) Find the lifting of the irreducible characters of Z_3 to A_4.

(e) Use (d) to complete the character table of A_4.