(10) Display the hilbert series of $\mathbf{R}^*$ and explain why it should be of the form

$$F_{\mathbf{R}^*}(q) = \prod_{i=1}^{3}(1+q+\cdots+q^{d_i-1})$$

(11) Construct a basis for the Harmonincs of RG.

(12) Compute a random homogeneous invariant I_6 of degree 6

(13) Construct the polynomial $Q(y_1, y_2, y_3)$ giving

$$I_6 = Q(I_1, I_2, I_3)$$

(a) For NGR

(1) Compute the Hilbert series of its ring of invariants $\mathbf{R}^{NGR}$.

(2) With a little manipulation you should be able to write it in the form

$$F_{\mathbf{R}^{GR}}(q) = \frac{1+q^{d_4}}{(1-q^{d_1})(1-q^{d_2})(1-q^{d_3})}$$

(3) Calculate the first 10 terms of this series.

(4) Explain what the exponents d_1, d_2, d_3, d_4 tell you about the basic systems of "*quasi-generators and separators*" that you may be able to construct for the ring $\mathbf{R}^{NGR}$.

(5) Construct three homogeneous GR-invariants I_1, I_2, I_3 of degrees d_1, d_2, d_3

(6) Verify that I_1, I_2, I_3 are algebraically independent by computing the Jacobian $\|\partial_{x_i} I_j\|_{i,j=1}^{4}$ (if not go back to (5)).

(6) Construct a homogeneous invariant η of degree d_4.

(7) Construct a polynomial $P(y_1, y_2, y_3, y)$ giving that

$$P(I_1, I_2, I_3, \eta) = 0$$

(8) Use this result to show that every element $f \in \mathbf{R}^{NGR}$ may be written in the form

$$f = P_o(I_1, I_2, I_3) + \eta\, P_1(I_1, I_2, I_3)$$

with P_o and P_1 uniquely determined by f. (Hint: Prove that 1 and η constitute a basis for the quotient $\mathbf{R}/(I1, I2, I3)$.

(8) Can you give a good reason why η turns out to be a multiple of the Jacobian $\|\partial_{x_i} I_j\|_{i,j=1}^{4}$? (Hint: Factor this Jacobian).