

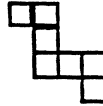
PART II

SYMMETRIC FUNCTION METHODS

Do 6 of the following 8 problems

S.1 Expand the monomial symmetric function $m_{321}(x_1, x_2, x_3)$ in terms of

- (a) The homogeneous basis
- (a) The elementary basis
- (a) The power basis
- (a) The Schur basis

S.2 Let D denote the following diagramGive the expansion of S_D in terms of

- (a) The Schur basis
- (b) The elementary basis
- (c) The monomial basis

S.3

- (a) Compute the scalar product

$$C_{321,321,42} = \frac{1}{6!} \sum_{\sigma \in S_6} \chi^{321}(\sigma) \chi^{321}(\sigma) \chi^{42}(\sigma)$$

- (b) Give a representation theoretical interpretation of the resulting integer.

S.4 Determine the smallest tensor power of the irreducible representation A^{42} of S_6 that contains at least one occurrence of the alternating representation of S_6 .