APPLIED ALGEBRA QUALIFIER May 31 2000

PART I

REPRESENTATION THEORY

Do 3 of the following 4 problems

R.1 Let G be a non abelian group of order p^3 with p prime. Show that G has p^2+p-1 conjugacy classes and p^2 distinct one dimensional representations

R.2 Use a representation theory argument to show that every group of prime order is abelian.

R.3 Let H be a subgroup of a finite group G and let

$$G \;=\; H\tau_1 H \;+\; H\tau_2 H \;+\; \cdots \;+\; H\tau_r H \qquad (\tau_1 = id)$$

be the double H-coset decomposition of G. Set

$$\theta = H/|H|$$

and let

$$\mathcal{G}_H \;=\; \{\theta f \theta \;:\; f \in \mathcal{A}(G)\}$$

be the subalgebra of $\mathcal{A}(G)$ generated by the elements

$$g_1 = \theta\,\tau_1\theta\,,\; g_2 = \theta\,\tau_2\theta\,,\; \cdots\,,\; g_r = \theta\,\tau_r\theta\,,$$

Show that $\mathcal{G}_H$ is commutative if and only if the action of G on the left ideal $\mathcal{A}(G)\,\theta$ is multiplicity-free. (Hint: Work with the Fourier transform of θ).

R.4 Suppose that every irreducible representation of a given finite group G is equivalent to a real representation. What can you say about the number of involutions of G?.